Vulnavia's Degree Network Cup

Summation of Series

Using the definition $\cot x = \frac{\cos x}{\sin x}$ and the series expansions for cos x and sin x:

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \frac{x^8}{8!} - \frac{x^{10}}{10!} + ...$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \frac{x^7}{7!} + \frac{x^9}{9!} - \frac{x^{11}}{11!} + ...$$

produce the series expansion for cot x in ascending powers of x.

From Mittag-Leffler's expansion theorem, cot x can be expanded as:

$$\cot x = \frac{1}{x} + 2x \left\{ \frac{1}{x^2 - \pi^2} + \frac{1}{x^2 - 4\pi^2} + \frac{1}{x^2 - 9\pi^2} + \frac{1}{x^2 - 16\pi^2} + ... \right\}$$

By applying the binomial theorem, term by term, obtain an expansion for cot x in ascending powers of x.

Compare the two series expansions for cot x.

By equating the coefficients of like terms, find the sum of each of the following series:

$$\sum_{n=1}^{n=\infty} \frac{1}{n^2}, \quad \sum_{n=1}^{n=\infty} \frac{1}{n^4}, \quad \sum_{n=1}^{n=\infty} \frac{1}{n^6}, \quad \sum_{n=1}^{n=\infty} \frac{1}{n^8}, \quad \sum_{n=1}^{n=\infty} \frac{1}{n^{10}} ....$$

Put the results in sequence and produce a generating formula for the summation:

$$\sum_{n=1}^{n=\infty} \frac{1}{n^{2p}} \qquad \text{(where } p = 1, 2, 3 .... \text{)}$$

Please register for entry by E-Mail: Chancellor@nightparklane.com