

The Vulnavian Degree Network Level 1

Limits and Series - Study Guide

Task 0284: Limits (12 pages)

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Workshop Task 0285: Limits (6 pages)

Task 0286: Calculus (14 pages)

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Workshop Task 0287:

Calculus w/s

(Not Included on the Standard Edit CD.)

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Limits and Series - Study Guide

Task 0288: Series (7 pages)

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Workshop Task 0289:

Series w/s

(Not Included on the Standard Edition CD.)

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Workshop Task 0292: (11 Pages)
L'Hopital's Rule / $(\sinh^{-1}x)^2$ Series

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END OF THE STUDY GUIDE FOR LIMITS AND SERIES