PERCENTAGES AND FINANCE

1. Give the multiplier for each percentage increase.

(a) 19% The multiplier = 1 + 19% = 1 + 0.19 = 1.19 (1 mark)

(b) 7% The multiplier = 1 + 7% = 1 + 0.07 = 1.07 (1 mark)

(c) 0.3% The multiplier = 1 + 0.3% = 1 + 0.003 = 1.003 (1 mark)

(d) 1.04% The multiplier = 1 + 1.04% = 1 + 0.0104 = 1.0104 (1 mark)

2. Jessica, who was on a salary of $ 29,500, was given a pay rise of 6%. What is her new salary?

The multiplier for a 6% increase = 1 + 6% = 1 + 0.06 = 1.06

The new salary = the old salary x the multiplier

= 29500 x 1.06

= 31270

Therefore, Jennifer's new salary = $ 31270 (3 marks)

3. In 2006 the population of a city was 2,836,000.
By 2012 its population had increased by 9%.
What was the population in 2012?

The multiplier for a 9% increase = 1 + 9% = 1 + 0.09 = 1.09

The new population = the old population x the multiplier

= 2836000 x 1.09

= 3091240

Therefore, the population in 2012 = 3,091,240 (3 marks)

4. Give the multiplier for each percentage decrease.

(a) 12% The multiplier = 1 - 12% = 1 - 0.12 = 0.88 (1 mark)

(b) 3% The multiplier = 1 - 3% = 1 - 0.03 = 0.97 (1 mark)

(c) 3.4% The multiplier = 1 - 3.4% = 1 - 0.034 = 0.966 (1 mark)

(d) 0.8% The multiplier = 1 - 0.8% = 1 - 0.008 = 0.992 (1 mark)

5. Decrease € 34 by 16%.

The multiplier for a 16% decrease = 1 - 16% = 1 - 0.16 = 0.84

The decreased amount = 34 x the multiplier = 34 x 0.84 = 28.56

Therefore, the decreased amount = € 28.56 (3 marks)

6. Marcus went to school with € 5.5. He spent 95 cents at the school shop.
What percentage of his pocket money did he spend at the school shop?

Calculate $\dfrac{\text{money spent at shop (in cents)}}{\text{original amount of money (in cents)}} \times 100\%$

$= \dfrac{95}{550} \times 100\% = 17.272727\ldots\% = 17.3\%$ (to 1 d.p.)

Therefore, the percentage of pocket money spent = 17.3 % (to 1 d.p.)

(3 marks)

[Alternatively:

Calculate $\dfrac{\text{money spent at shop (in euros)}}{\text{original amount of money (in euros)}} \times 100\%$

$= \dfrac{0.95}{5.5} \times 100\% = 17.272727\ldots\% = 17.3\%$ (to 1 d.p.)

… as before.]

7. As a result of "the crisis" the number of students in a school dropped from 620 to 480.
What was the percentage decrease?

The percentage **change** $= \dfrac{\text{new amount - original amount}}{\text{original amount}} \times 100\%$

$= \left(\dfrac{480 - 620}{620}\right) \times 100\%$

$= \dfrac{-140}{620} \times 100\%$

$= -22.580645\ldots\%$

Therefore, the percentage **decrease** = 22.6 % (to 1 d.p.) (3 marks)

8. Edward bought a painting for £ 560 but had to sell it for only £ 364.
What was the percentage loss?

The percentage **change** $= \dfrac{\text{new amount - original amount}}{\text{original amount}} \times 100\%$

$= \left(\dfrac{364 - 560}{560}\right) \times 100\%$

$= \dfrac{-196}{560} \times 100\%$

$= -35\%$

Therefore, the percentage **loss** = 35 % (3 marks)

9. A bank pays 0.4% simple interest on the money that each saver keeps in the bank for a year. Paula keeps $ 862 in the bank for five years.

How much money will she have in the bank from this investment after 5 years?

The interest paid each year = $\frac{0.4}{100}$ x 862 = 3.448

Five payments = 5 x 3.448 = 17.24

Add this extra 17.24 to the original 862 to get 879.24

Therefore, the value of the investment after five years = $ 879.24

(3 marks)

10. David invests € 9000 at 2% compound interest for seven years.
What is the total amount of this investment after seven years?

Use the compound interest formula: $T = P \times (m)^n$

(where T is the new total, P is the principal amount (starting amount), m is the multiplier and n is the number of years.)

The multiplier for an increase of 2% = 1 + 0.02 = 1.02

So the total amount = $9000 \times (1.02)^7$

= 10338.1710088 ….

Therefore, after 7 years the investment = € 10338.17 (to 2 d.p.)

(4 marks)

11. Marta invests € 6000 at 3.2% compound interest for four years.
What is the overall percentage increase?

Method 1: Use the compound interest formula: $T = P \times (m)^n$
with P = 6000, m = 1 + 0.032 = 1.032 and n = 4.

So after four years, the total amount = $6000 \times (1.032)^4$

The percentage increase = ($\frac{\text{new amount – original amount}}{\text{original amount}}$) x 100%

$= \left(\frac{6000\,(1.032)^4 - 6000}{6000} \right) \times 100\%$

= 13.4276 ... %

Therefore, the overall percentage increase = 13.43 % (to 2 d.p.) (6 marks)

11. Continued …

Method 2:

From the working out in Method 1,

the percentage increase = $\left(\frac{6000\,(1.032)^4 - 6000}{6000} \right) \times 100\%$

But dividing throughout by the principal value 6000 leaves:

the percentage increase = $\{ (1.032)^4 - 1 \} \times 100\%$

(In general, the % increase = (multiplier ^{n} - 1) x 100%)

So the percentage increase = 13.4276 ... % as before.

= 13.43 % (to 2 d.p.)

12. The number of workers in a call centre fell by 15% to 153.
How many workers were there originally?

A decrease has occurred so use the multiplier for a percentage decrease.

The multiplier for a decrease of 15% = 1 - 15% = 1 - 0.15 = 0.85

The final amount = the original amount x the multiplier

Rearrange to get: the original amount = $\frac{\text{the final amount}}{\text{the multiplier}}$

$= \frac{153}{0.85} = 180$

Therefore, the original number of workers = 180 (3 marks)

13. In a sale the price of a mobile phone is reduced to \$ 185.
This is a 26% reduction on the original price.
What was the original price?

A decrease has occurred so use the multiplier for a percentage decrease.

The multiplier for a decrease of 26% = 1 - 26% = 1 - 0.26 = 0.74

The final amount = the original amount x the multiplier

Rearrange to get: the original amount = $\frac{\text{the final amount}}{\text{the multiplier}}$

$= \frac{185}{0.74} = 250$

Therefore, the original price = \$ 250 (3 marks)

14. Rachael invested some money at 5% compound interest for two years. After 2 years she had £ 23373 in the bank.

How much did she invest originally?

Use the compound interest formula: $T = P \times (m)^n$

(where T is the new total, P is the principal amount (starting amount), m is the multiplier and n is the number of years.)

The multiplier for an increase of 5% = 1 + 0.05 = 1.05

So the total amount = $P \times (1.05)^2$

= 23373 (given)

To find P, solve the equation: $(1.05)^2 P = 23373$ (3 marks)

Divide both sides by $(1.05)^2$ to get: $P = \frac{23373}{(1.05)^2}$

= 21200 (2 marks)

Therefore, the original amount invested = £ 21200 (Total = 5 marks)

(Total for Percentages & Finance = 50 marks)

RATIO

1. In a plane the seats are allocated as business class and economy class in the ratio 3 : 23

(a) What fraction of the seats are business class?

The total number of “parts” = 3 + 23 = 26

Of these 26 parts, 3 are for business class.

So the fraction of the seats that are business class = $\frac{3}{26}$ (2 marks)

(b) If there are 234 seats altogether, how many are economy class?

The fraction of the seats that are economy class = $\frac{23}{26}$

So the number of seats that are economy class = $\frac{23}{26} \times 234 = 207$

Therefore, the number of seats that are economy class = 207 (3 marks)

2. A map has a scale of 8 cm to 5 km.

(a) Rewrite the scale as a ratio in its simplest form.

Start by finding the number of centimetres in 1 kilometre.

1 km = 1000 m = 100 x 1000 cm = 100 000 cm.

So the scale of the map is equivalent to: 8 cm to 500 000 cm

The scale as a ratio is: 8 : 500 000

Divide through by 8 to get this ratio in its simplest form.

The ratio becomes, 1 : 62 500

Therefore, the scale of the map in its simplest form is, 1 : 62 500 (3 marks)

(b) How long is a path that measures 0.6 cm on the map?

Use the unitary method.

Since 8 cm represents 5 km

then 1 cm represents $\frac{5}{8}$ km

So, 0.6 cm represents 0.6 x $\frac{5}{8}$ km = 0.375 km ≡ 375 m

Therefore, the length of the path = 375 m (3 marks)

(c) How long should an 875 m road be on the map?

Use the unitary method.

Since 8 cm represents 5 km

then $\frac{8}{5}$ cm represents 1 km

Now, 875 m = 0.875 km

So, 0.875 km is represented by 0.875 x $\frac{8}{5}$ cm = 1.4 cm

Therefore, the length of the road on the map = 14 mm (3 marks)

3. The ratio of male to female spectators at a football match is 11 : 3
   22935 males watched the match.

   What was the total attendance at the game?

The total number of "parts" = 11 + 3 = 14

Now, $\frac{11}{14}$ of the spectators are male.

Let the total attendance be represented by T.

So, $\frac{11}{14}T = 22935$

Multiply through by 14 to get: 11T = 321090

Divide through by 4, T = 29190

Therefore, the total attendance at the game = 29190 (3 marks)

4. Prices are up in the ratio 23 : 20
   What percentage increase is this?

   Divide the ratio through by 20 to get, $\frac{23}{20} : 1$

   So the multiplier for the percentage increase = $\frac{23}{20} = 1.15$

   Since the multiplier = 1 + % increase, then the % increase = 15%

   Therefore, the ratio represents a percentage increase of 15% (3 marks)

(Total for Ratio = 20 marks)

[Note: The test was split at this point.]

SPEED

1. A train travels a distance of 228 km at an average speed of 120 km/h.

(a) How long does the journey take?

From $\text{speed} = \frac{\text{distance}}{\text{time}}$, $\text{time} = \frac{\text{distance}}{\text{speed}}$

So the time taken $= \frac{228}{120} = 1.9$

Therefore, the journey takes 1.9 hours. (2 marks)

(b) If the train started the journey at 7.40 am, at what time did it reach its destination?

Convert 1.9 hours into hours and minutes.

0.9 hours = 60 x 0.9 minutes = 54 minutes.

So 1.9 hours = 1 hour 54 minutes

Add the hour to 7.40 am to get 8.40 am
Add 20 minutes to get to 9 am
Add the remaining 34 minutes to get: 9.34 am

Therefore, the train arrived at 9.34 am (3 marks)

2. How long will it take an athlete to run 1350 metres at an average speed of 6 m/s? (Give the answer in minutes and seconds.)

From $\text{speed} = \frac{\text{distance}}{\text{time}}$, $\text{time} = \frac{\text{distance}}{\text{speed}}$

So the time taken $= \frac{1350}{6} = 225$ (seconds) (2 marks)

Now convert the answer in seconds to minutes and seconds.

$\frac{225}{60} = 3.75$ So there are 3 whole minutes in 225 seconds.

3 x 60 = 180 seconds

So the remainder = 225 - 180 = 45 (seconds) (2 marks)

Therefore, the time taken to run 1350 m = 3 minutes 45 seconds

(Total = 4 marks)

3. Convert 25 m/s into km/h.

25 metres per second ≡ $\frac{25}{1000}$ km per second

But as the number of seconds in 1 hour = 60 x 60 = 3600,

then 0.025 km per second ≡ 0.025 x 60 x 60 km per second

= 90 km / h (2 marks)

[Note: To convert from m/s to km/h, multiply the number in m/s by 3.6]

4. A car travels at 82 km/h for 4 hours, then it slows down to do the last 72 minutes of the journey at 30 km/h.

(a) What is the total distance of this journey?

From speed = $\frac{\text{distance}}{\text{time}}$, distance = speed x time

So the distance for the first part of the journey = 82 x 4 = 328 km

Note that 72 minutes = 1 hour + $\frac{12}{60}$ of an hour

= 1hour + $\frac{1}{5}$ hour = 1.2 hours

So the distance for the final part of the journey = 30 x 1.2 = 36 km

The total distance = 328 + 36 = 364

Therefore, the total distance = 364 km (3 marks)

(b) What is the average speed of the car over the whole journey?

Speed = $\frac{\text{distance}}{\text{time}}$

From part (a), the total distance = 364 km

The total time = 5 hours 12 minutes = 5.2 hours

So, the average speed = $\frac{364}{5.2}$ = 70

Therefore, the average speed = 70 km/h (3 marks)

(Total for Speed = 17 marks)

RATES

1. Marina says that she can read a book with 301 pages in 7 hours.

(a) What is her rate of reading in pages / hour ?

Divide 301 by 7 to get the result.

$$\frac{301}{7} = 43$$

Therefore, Marina can read 43 pages / hour. (1 mark)

(b) How long would she take to read 387 pages at this rate?

Use the unitary method.

She reads 301 pages in 7 hours

So she reads 1 page in $\frac{7}{301}$ hours

So she reads 387 pages in $387 \times \frac{7}{301}$ hours = 9 hours

Therefore, the time taken to read 387 pages = 9 hours. (2 marks)

(c) How many **seconds** does she take to read one page?

She reads 301 pages in 7 hours

So she reads 1 page in $\frac{7}{301}$ hours

Convert $\frac{7}{301}$ hours into seconds by multiplying by (60 x 60)

So, the time taken = $\frac{7}{301} \times 60 \times 60 = 83.7209 \ldots$ (seconds)

Therefore, the time taken to read one page = 83.7 seconds (to 3 s.f.)

(2 marks)

2. Eighteen maths textbooks cost € 441.

(a) How much will 24 maths textbooks cost at the same price?

Use the unitary method.

18 maths textbooks cost € 441

So, 1 maths textbook costs € $\frac{441}{18}$

So, 24 maths textbooks cost € $\frac{441}{18}$ x 24 = € 588

Therefore, the cost of 24 maths textbooks = € 588 (2 marks)

(b) How many of these maths textbooks can be bought for € 700 ?

The cost of one maths textbook = € $\frac{441}{18}$ = € 24.5

So the number of maths textbooks that can be bought for € 700 = $\frac{700}{24.5}$

= 28.571 …

Therefore, 28 maths textbooks can be bought for € 700 (2 marks)

3. It takes a photocopier 18 seconds to produce 15 copies.

How long will it take to produce 25 copies at the same rate?

Use the unitary method.

15 copies are produced in 18 seconds

So, 1 copy is produced in $\frac{18}{15}$ seconds

So, 25 copies are produced in 25 x $\frac{18}{15}$ seconds = 30 seconds

Therefore, the time taken to produce 25 copies = 30 seconds (2 marks)

(Total for Rates = 11 marks)

VARIATION

1. Y is directly proportional to R.

(a) If Y = 96 when R = 64, find Y when R = 24

Set up the variation relationship: $Y \alpha R$

Convert this relationship into an equation by introducing a constant of proportionality k.

So, Y = kR

Use the given information to find the value of k.

Set Y = 96 and R = 64 in the equation.

So, 96 = 64k

Divide through by 64 to get: $k = \frac{96}{64} = 1.5$

So the general equation is: Y = 1.5 R (2 marks)

Now set R = 24 in this equation to find the required value of Y.

Thus, Y = 1.5 x 24 = 36

Therefore, Y = 36 when R = 24. (1 mark) (Total = 3 marks)

(b) R when Y = 45

From part (a), Y = 1.5 R

Set Y = 45 and find R.

So, 1.5 R = 45

Divide through by 1.5 to get: $R = \frac{45}{1.5} = 30$

Therefore, R = 30 when Y = 45 (1 mark)

2. The cost, in euros, of a trip varies directly with the square root of the number of miles travelled.

The cost of a 729-mile trip is 135 euros.

(a) What is the cost of a 500-mile trip? (To the nearest euro.)

Let C = the cost of the trip in euros
and M = the number of miles travelled

Set up the variation relationship: $C \propto \sqrt{M}$

Convert this relationship into an equation by introducing a constant of proportionality k.

So, $C = k\sqrt{M}$

Use the given information to find the value of k.

Set C = 135 and M = 729 in the equation.

So, $135 = k \times \sqrt{729} = 27k$ Thus, $27k = 135$

Divide through by 27 to get: $k = \frac{135}{27} = 5$

So the general equation is: $C = 5\sqrt{M}$ (2 marks)

Now set M = 500 in this equation to find the required value of C.

Thus, $C = 5 \times \sqrt{500} = 111..8033$ ….

Therefore, the cost of a 500-mile trip = € 112 (to the nearest euro).

(2 marks) (Total = 4 marks)

(b) What is the distance of a trip costing 220 euros?

From part (a), $C = 5\sqrt{M}$

Set C = 220 and rearrange the equation to find the required value of M.

$5\sqrt{M} = 220$

Divide through by 5 to get: $\sqrt{M} = \frac{220}{5} = 44$

Thus, $\sqrt{M} = 44$

Square both sides to get: $M = 44^2 = 1936$

Therefore, the distance of a trip costing € 220 is 1936 miles. (2 marks)

3. y is inversely proportional to the cube root of x.

(a) If y = 3 when x = 216 find y when x = 125

Set up the variation relationship: $y \propto \frac{1}{\sqrt[3]{x}}$

Convert this relationship into an equation by introducing a constant of proportionality k.

So, $y = k\frac{1}{\sqrt[3]{x}}$

Use the given information to find the value of k.

Set y = 3 and x = 216 in the equation.

So, $3 = k\frac{1}{\sqrt[3]{216}} = k\left(\frac{1}{6}\right)$ Thus, $3 = \frac{k}{6}$

Multiply through by 6 to get: k = 18

So the general equation is: $y = \frac{18}{\sqrt[3]{x}}$ (3 marks)

Now set x = 125 in this equation to find the required value of y.

Thus, $y = \frac{18}{\sqrt[3]{125}} = \frac{18}{5} \equiv 3.6$

Therefore, when x = 125, y = 3.6 (2 marks)

(Total = 5 marks)

(b) x when y = 6

From part (a), $y = \frac{18}{\sqrt[3]{x}}$

Set y = 6 and rearrange the equation to find the required value of x.

$6 = \frac{18}{\sqrt[3]{x}}$

Swap around $\sqrt[3]{x}$ and 6 to get: $\sqrt[3]{x} = \frac{18}{6}$ Thus, $\sqrt[3]{x} = 3$

Cube both sides to get: $x = 3^3 = 27$

Therefore, when y = 6, the value of x is 27 (2 marks)

4. The grant available to a group of students was inversely proportional to the number of students.

When 40 students needed a grant they received $ 60 each.

(a) What would the grant have been if 30 students had needed one?

Let G = the grant in dollars and N = the number of students.

Set up the variation relationship: $G \propto \frac{1}{N}$

Convert this relationship into an equation by introducing a constant of proportionality k.

So, $G = k\frac{1}{N}$

Use the given information to find the value of k.

Set G = 60 and N = 40 in the equation.

So, $60 = k \times \frac{1}{40}$

Multiply through by 40 to get: k = 2400

So the general equation is: $G = \frac{2400}{N}$ (2 marks)

Now set N = 30 in this equation to find the required value of G.

Thus, $G = \frac{2400}{30} = 80$

Therefore, if 30 students need a grant, the grant would be $ 80

(1 mark) (Total = 3 marks)

(b) If the grant had been $ 37.50 each, how many students would have needed it?

From part (a), $G = \frac{2400}{N}$ Rearrange to get: $N = \frac{2400}{G}$

Set G = 37.5 to find the required value of N

So, $N = \frac{2400}{37.5} = 64$

Therefore, 64 students would have had a grant of $ 37.50 each (2 marks)

(Total for Variation = 22 marks)

GRAND TOTAL = 120 MARKS

END OF THE MATRIX 2 TEST TUTORIAL