

The Vulnavian Degree Network Level 3

Partial Differential Equations - Study Guide

TASK 0311r (11 pages)

Review of Partial Differentiation: The Basics

PARTIAL DIFFERENTIAL EQUATIONS COURSE

TASK 0311 (8 pages)

1. **Introduction** (Pg. 1)
2. **Second Order Constant Coefficient Equations** (Pg. 2)
3. **Partial Differential Equation Classification:** (Pg. 2)
Elliptic Type / Parabolic Type / Hyperbolic Type
Investigation 20: "Phantom Conics" (See 0311c)
4. **Euler's Equation** (Pg. 3)
5. **General Solution of Euler's Equation** (Pg. 3)
Elliptic & Hyperbolic Types
Parabolic Type (Pg. 6)
6. **Euler's Equation: Examples** (Pg. 7)
Example 1: Laplace's equation in two variables:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

Example 2: $3 \frac{\partial^2 u}{\partial x^2} + 4 \frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 u}{\partial y^2} = 0$ (Pg. 7)

Example 3: $16 \frac{\partial^2 u}{\partial x^2} + 8 \frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 u}{\partial y^2} = 0$ (Pg. 8)

PARTIAL DIFFERENTIAL EQUATIONS:

SEPARATION OF VARIABLES: TASK 0312 (22 pages)

1. **Separation of Variables: The Wave Equation** (Pg. 1)
Boundary Conditions (Pg. 3)
2. **The Heat Conduction Equation** (Pg. 6)
Boundary Conditions (Pg. 7)
Altered Boundary Conditions (Pg. 9)
3. **The Heat Conduction Equation: Examples** (Pg.11)
Example 1 (Pg.11)
Example 2 (Pg.13)
4. **Partial Differential Equations in 2D or 3D** (Pg.16)
The Laplacian Operator
5. **Laplace's Equation in 2D** (Pg.17)
6. **Laplace's Equation in 2D: Example** (Pg.21)

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Partial Differential Equations Workshop 0313 (13 pages)

Partial Differential Equations: (Pg. 1)

Question 1: $\frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \frac{\partial R}{\partial r}) = \frac{1}{c^2} \frac{\partial^2 R}{\partial t^2}$

Question 2: $r^2 \frac{\partial^2 V}{\partial r^2} + r \frac{\partial V}{\partial r} + \frac{\partial^2 V}{\partial \theta^2} = 0$ (Pg. 2)

Question 3: $\frac{\partial^2 V}{\partial x^2} = \frac{\partial V}{\partial y}$ (Pg. 3)

Question 4a: $\frac{\partial^2 Z}{\partial x^2} = \frac{2}{a} \frac{\partial Z}{\partial t} + Z$ (Pg. 4)

Question 4b: $\frac{\partial Z}{\partial x} = -C \sec L \cos(L - x) \exp(-at)$ (Pg. 5)

Question 5a: $\frac{\partial^2 F}{\partial r^2} + \frac{2}{r} \frac{\partial F}{\partial r} = \frac{1}{c^2} \frac{\partial^2 F}{\partial t^2}$ (Pg. 6)

Question 5b: Proof of the Solution: (Pg. 6)
 $F(r, t) = \frac{1}{r} (A \cos pr + B \sin pr) \cos \omega t$

Question 6: $\frac{\partial^2 V}{\partial t^2} + V - 4 \frac{\partial V}{\partial x} = 0$ (Pg. 8)

Question 7: $\frac{1}{c^2} \frac{\partial^2 V}{\partial t^2} = \frac{\partial^2 V}{\partial r^2} + \frac{1}{r} \frac{\partial V}{\partial r}$ (Pg. 10)

Question 7: Proof: (Pg. 11)
Justification that $J_0(kr)$ is a solution of Bessel's equation with $\nu = 0$.

Question 7: Proof: (Pg. 12)
Justification that $J_0(kr)$ is a solution of Bessel's equation with $\nu = 0$:
 $J_0(kr)$ given in Sigma Form.

**END OF THE STUDY GUIDE FOR
PARTIAL DIFFERENTIAL EQUATIONS.**