

# The Vulnavian Degree Network Level 2

## Partial Differentiation - Study Guide

### PARTIAL DIFFERENTIATION: INTRODUCTORY COURSE

Task 0293 (30 pages)

(Culminating in the Derivation of Laplace's Equation in Polar Coordinates.)

1. **Functions of Several Independent Variables** Pg. 2
2. **Continuity of Functions** Pg. 3
3. **First Partial Derivatives** Pg. 4
  - Example 1:  $f_1(x, y) = x^2 - 4y^2$  Pg. 5
  - Example 2:  $f(x, y) = x + y \log_e x$  Pg. 6
  - Example 3:  $f(r, \theta) = r \cos \theta$  Pg. 6
4. **Functions of n Independent Variables** Pg. 6
5. **Function of a Function** Pg. 7
  - Example 1:  $f(x, y) = \tan^{-1}(y/x)$  Pg. 7
  - Example 2:  $f(u) = \cos u$  Pg. 8
6. **Higher Partial Derivatives** Pg. 9 -10
7. **Total Derivatives (From First Principles)** Pg.10-11
  - Example:  $f(x, y) = x^2 + y^2$  and  $x = \cosh t, y = t^2$  Pg.12
8. **Implicit Differentiation** Pg.12
  - Example 1a:  $f(x, y) = \tan^{-1} \frac{y}{x}$  and  $y = \cos x$  Pg.13
  - Example 1b: Check Ex.1a by Substitution Pg.14
  - Finding  $\frac{dy}{dx}$  &  $\frac{dz}{dy}$  when  $F(x,y,z) = 0, G(x,y,z) = 0$  Pg.15-16
9. **Higher Total Derivatives** Pg.18-19
  - INVESTIGATION 10: "Wrong Way Round"** Pg.18
  - The \*D Operator** Pg.19
10. **Total Derivatives: Further Consideration** Pg.20
  - Example 1:  $f(x, y, z) = (\sin^2 x) \log_e y + z^2$  Pg.20
  - Practical Example 1: Pendulum Pg.20
  - Practical Example 2: Gas Law Pg.21
11. **Homogeneous Functions** Pg.23
  - Example 1:  $f(x, y, z) = x^2 + y^2 - z^2 + xy$
  - Example 2:  $f(x, y) = \frac{x^2 + y^2}{4xy} + \cos\left(\frac{y}{x}\right)$
12. **Euler's Theorem: Proof** Pg.24
  - Example:  $f(x, y, z) = x^2 + y^2 - z^2 + xy$  Pg.25
13. **Change of Variables** Pg.26
  - Example 1  $(x, y) \rightarrow (s, t)$  Pg.26
  - Example 2  $(x, y) \rightarrow (r, \theta)$  Pg.27
14. **Laplace's Equation in Polar Coordinates (r,  $\theta$ )** Pg.28-30

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### Task 0294: Taylor's Theorem (4 pages)

1. **Taylor's Theorem for Functions of Two Independent Variables** (Pg.1)  
**INVESTIGATION 11: "The Lagrange Term"**
2. **Proof of the Series Expansion for the Function  $f(a + h, b + k)$**  (Pg.2)
3. **Taylor's Series Using Sigma Notation** (Pg.2)
4. **Alternative Form of Taylor's Series** (Pg.2)
5. **Taylor's Series for Functions of Two Independent Variables:**  
Example 1:  $f(x, y) = \cos xy$  (Pg.3)  
Example 2: Gas Law Application (Pg.4)

### Task 0295: Stationary Points in 2-D (7 pages)

1. **Maxima and Minima of Functions of Two Variables.** (Pg.1)
2. **Maximum Condition.** (Pg.1)  
**Condition for a Point  $(x, y, z)$  to be a Stationary Point** (Pg.1)
3. **Minimum Condition.** (Pg.2)  
**Condition for a Stationary Point.** (Pg.2)
4. **How to Determine the Nature of a Stationary Point Using Taylor's Series.** (Pg.3)
5. **Definition of Delta:** (Pg.4)  
$$\Delta = f^2_{xy}(a,b) - f_{xx}(a,b)f_{yy}(a,b)$$
6. **Saddle Points** (Pg.5)  
**INVESTIGATION 12:** (Pg.5)  
"Delta Zero" - Investigate  $\Delta = 0$  cases.
7. **Saddle Points: Contour Plot** (Pg.6)
8. **Stationary Points:** (Pg.6)  
Example 1:  $f(x, y) = \frac{1}{2}x^2 + y^2 + xy + x$

### Task 0295x: Stationary Points in 2-D (4 pages)

8. **Stationary Points:** (Pg.1)  
Example 2:  $f(x, y) = \frac{x^2y^2}{4} - x^2 - \frac{y^2}{2} + 1$   
Example 3:  $f(x, y) = xy^2 - 2xy + 2x^2 - 3x$  (Pg.3)

### **INVESTIGATION 13: "Inflexion Surfaces (Pg.4)**

### Workshop Task 0296: Stationary Points in 2-D

( Not Included on the Standard Edition CD.)

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**Task 0297:** Conditional Points and Lagrange Multipliers (9 pages)

1. **Conditional Stationary Points** (Pg.1)  
Example 1:  $f(x, y, z) = x + y^2 + z^2$   
Example 2:  $f(x, y, z) = x^2 y^2 z^2$  (Pg.2)  
Contour Plot (Pg.3)
2. **Lagrange Method of Undetermined Multipliers** (Pg.5)
3. Example 1: (Pg.6)  
Minimum Distance from (0, 0) to a curve.
4. Example 2: (Pg.8)  
Minimum Distance from (0, 0) to the Rectangular Hyperbola  $xy = c^2$

**END OF THE STUDY GUIDE FOR PARTIAL DIFFERENTIATION**