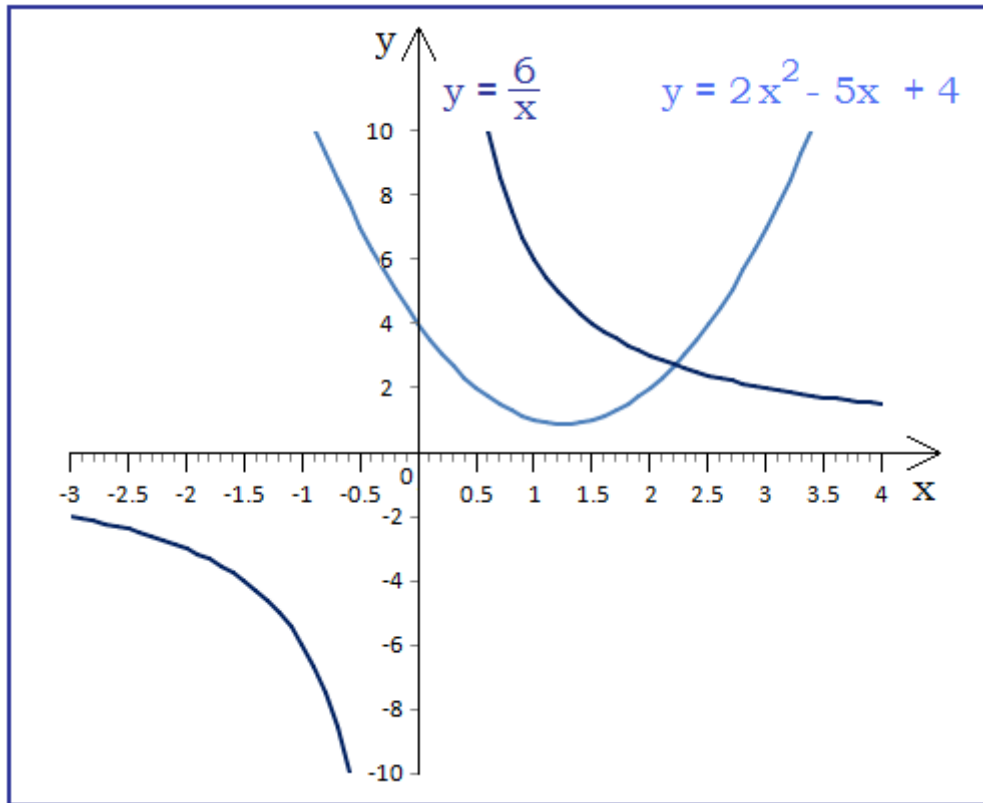


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Iterative Formulae

Raindrop 6

The following diagram shows the graphs of the parabola $y = 2x^2 - 5x + 4$ and the hyperbola $y = \frac{6}{x}$.



(i) Use the given graphs to find the number of real roots of the cubic equation: $2x^3 - 5x^2 + 4x - 6 = 0$

(ii) Show that this cubic equation has a real root α between the values of $x = 2.2$ and $x = 2.3$.

(iii) Use the iterative formula:
$$x_{n+1} = \frac{5}{\left(2 + \frac{4}{x_n^2} - \frac{6}{x_n^3}\right)}$$

to find the value of the root α correct to 9 decimal places.

The answers follow on the next page ...

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Raindrop 6

(i) Answer: At intersection points: $2x^2 - 5x + 4 = \frac{6}{x}$

Rearrange to get the cubic equation: $2x^3 - 5x^2 + 4x - 6 = 0$

From the graphs, since the hyperbola $y = \frac{6}{x}$ cannot cross the y-axis, there is only one intersection point.

So the given cubic equation only has one real root.

(ii) Answer

$$\text{Set } f(x) = 2x^3 - 5x^2 + 4x - 6$$

$$\text{Note, } f(2.2) = -0.104$$

$$\text{and } f(2.3) = +1.084$$

Since $f(x)$ changes sign between $x = 2.2$ and $x = 2.3$ there must be a value of x between these two values for which $f(x) = 0$. This value of x is the root $x = \alpha$.

(iii) Answer $\alpha = 2.209355137$ {to 9 d.p.}